Pascal's Triangle

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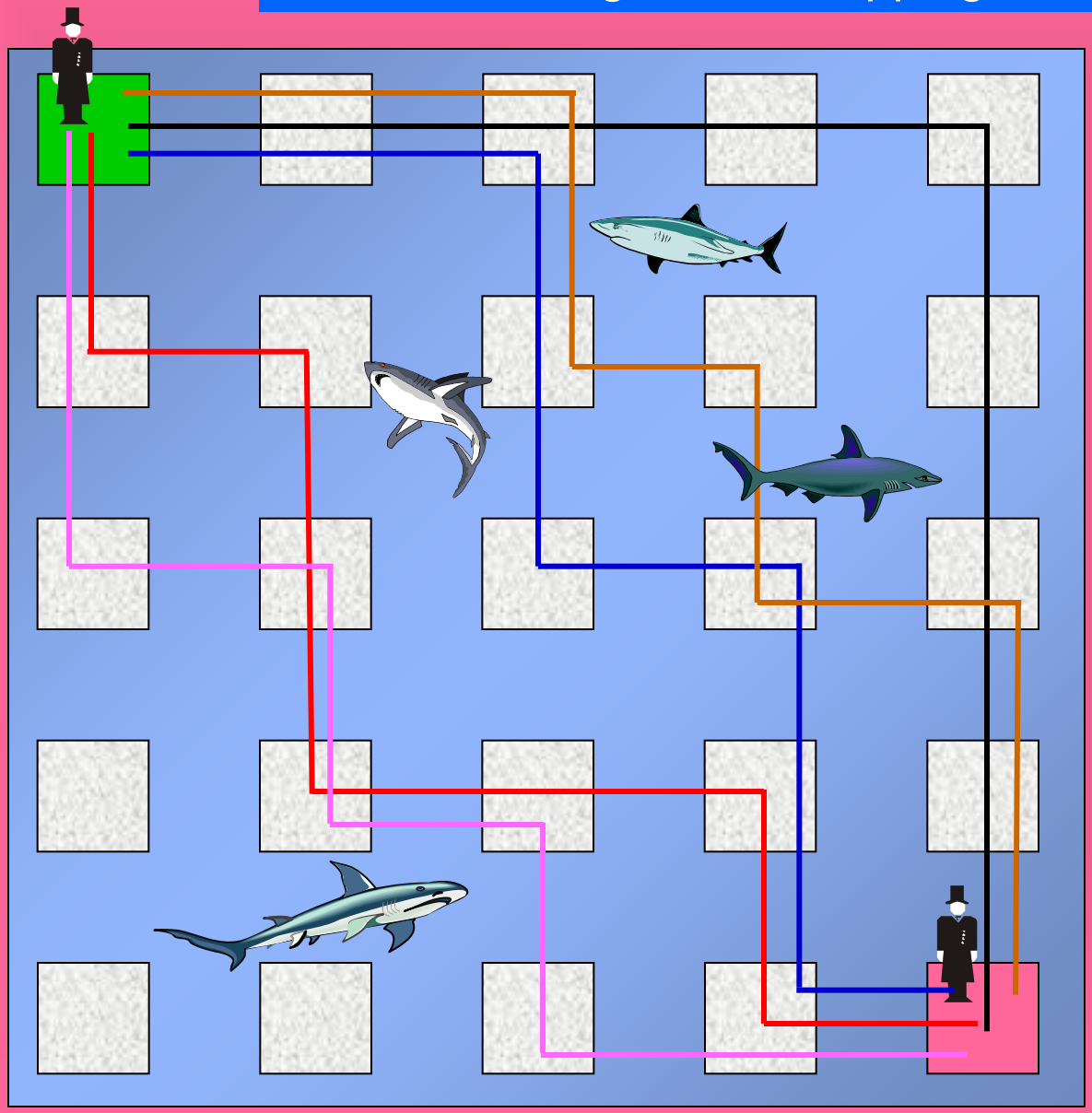
http://cs.txstate.edu/~ch04/webtest/itest/curr/curr2/math/PascalsTriangleLaCosta.ppt

Pascal's Triangle: The Stepping Stone Game



How many different routes are there from the Start stone to the Finish stone?

Rules:

You can only walk

East →

or

South ↓

from any stone.

We will start by looking at 5 possible routes (be careful how you walk)

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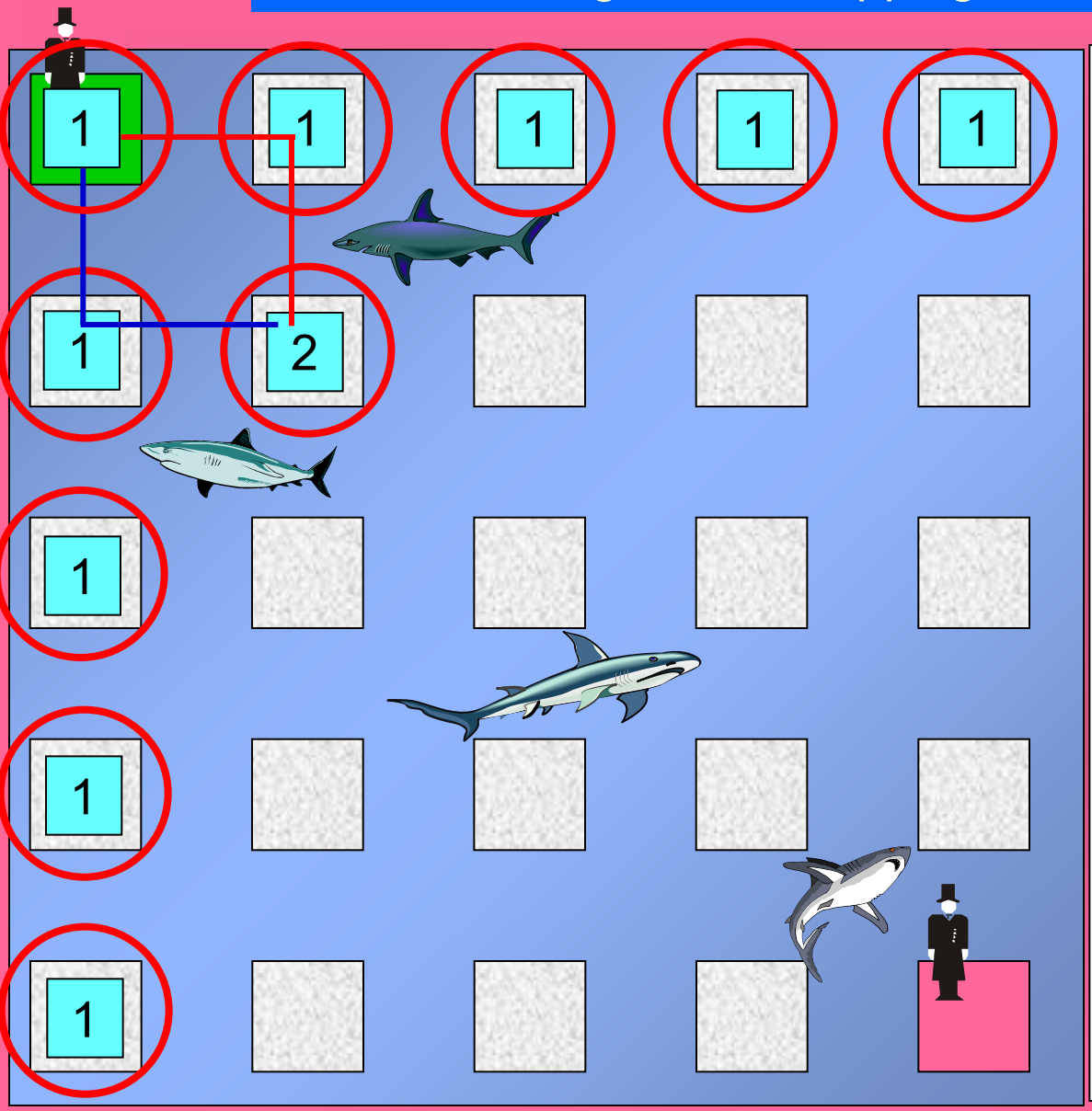

How many routes are there to:

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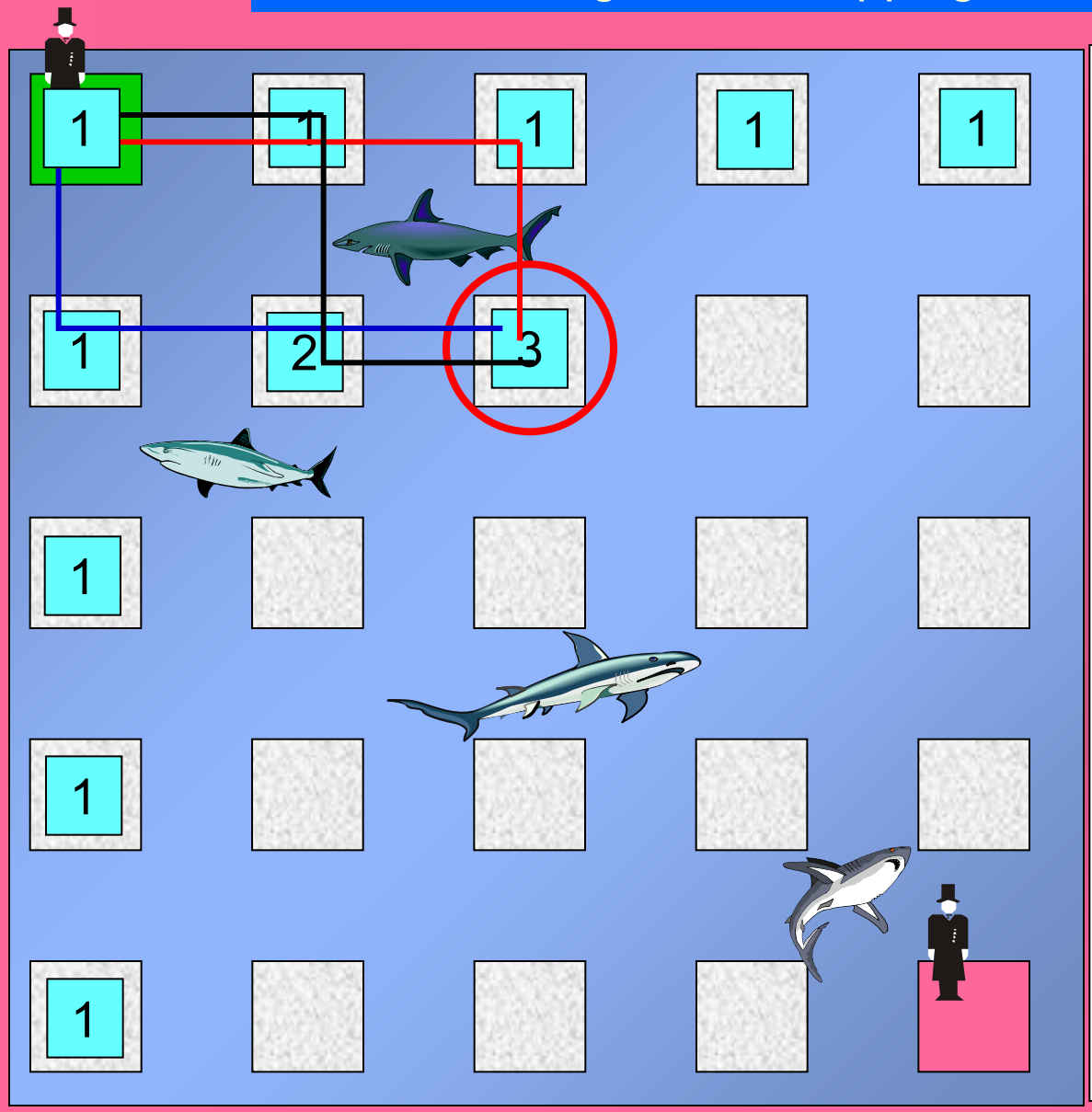

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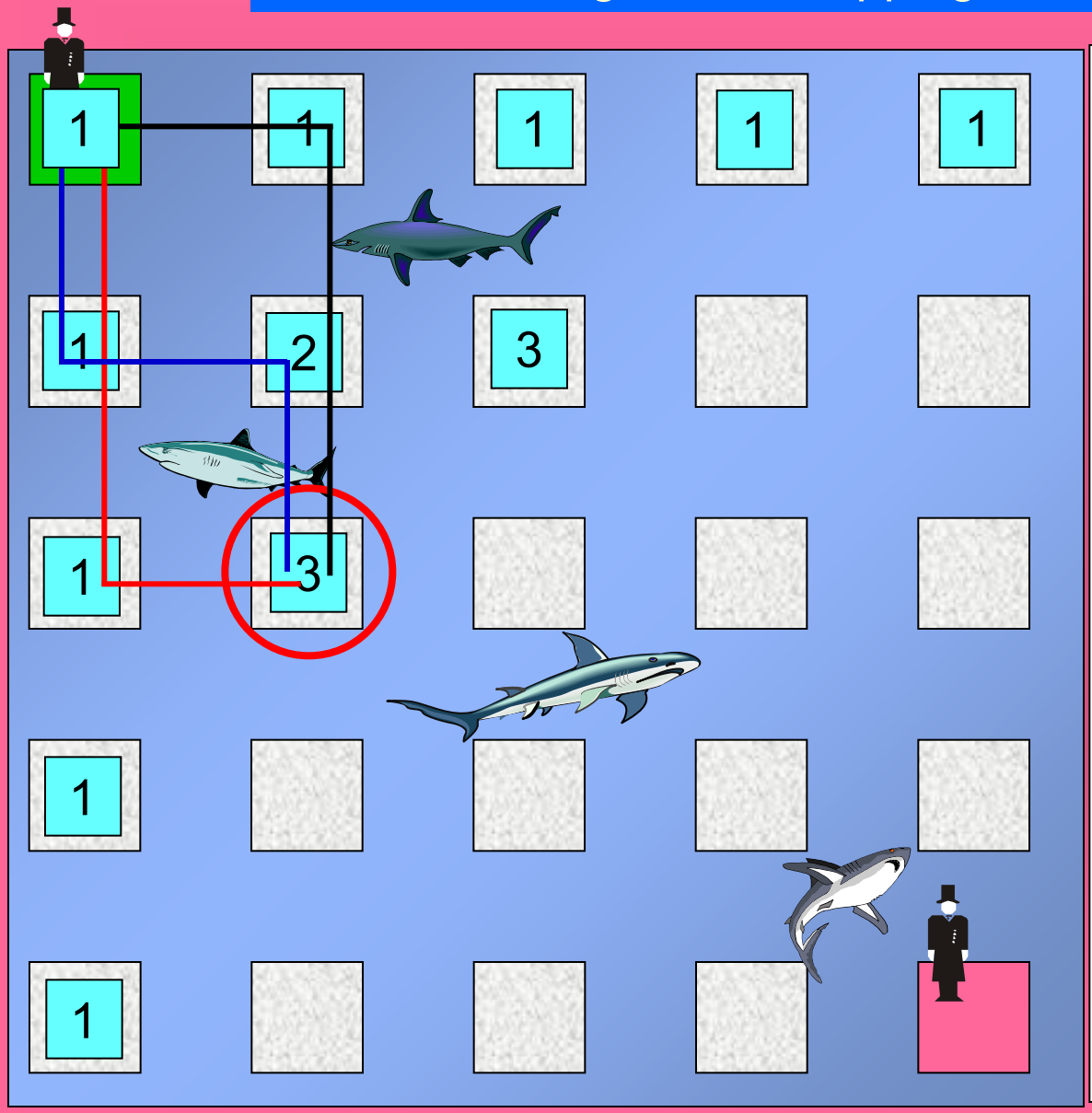

How many routes are there to:

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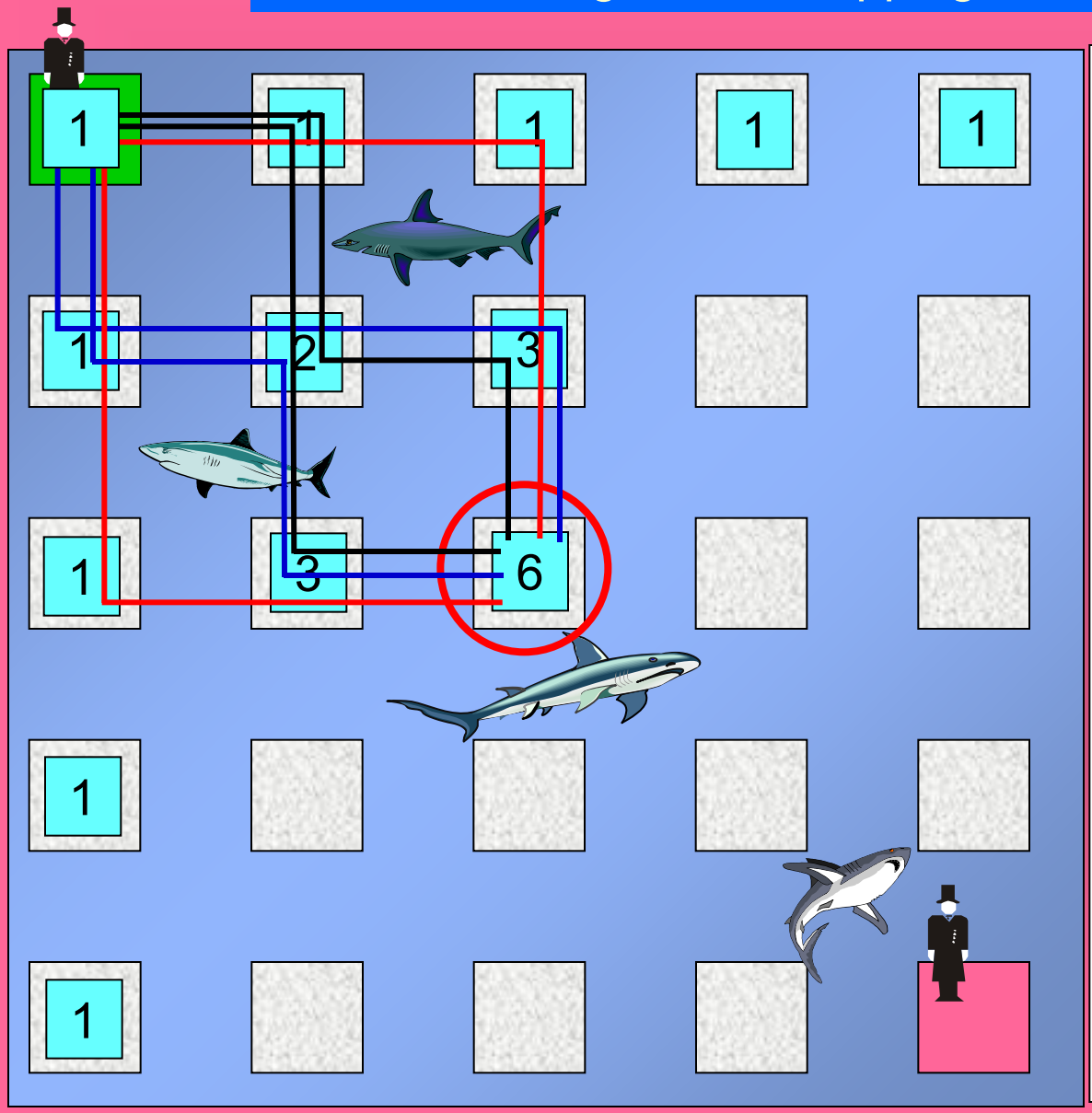

How many routes are there to:

Can you see all 6 of the routes?

How could you have calculated the 6 routes without the need to draw or visualise them?

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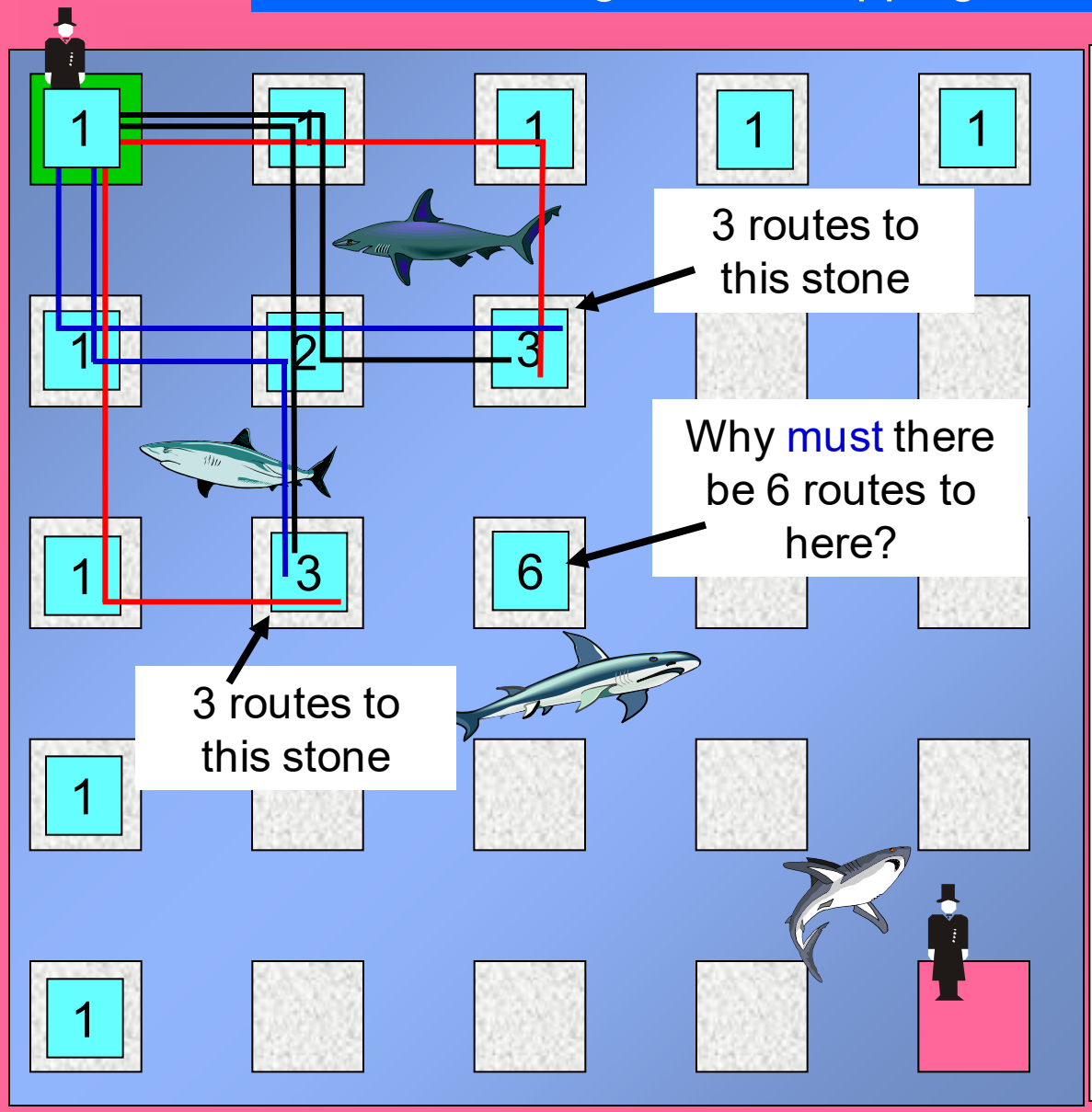

How many routes are there to:

Can you see all 6 of the routes?

How could you have calculated the 6 routes without the need to draw or visualise them?

What do you have to do to get the number of routes to any stone?

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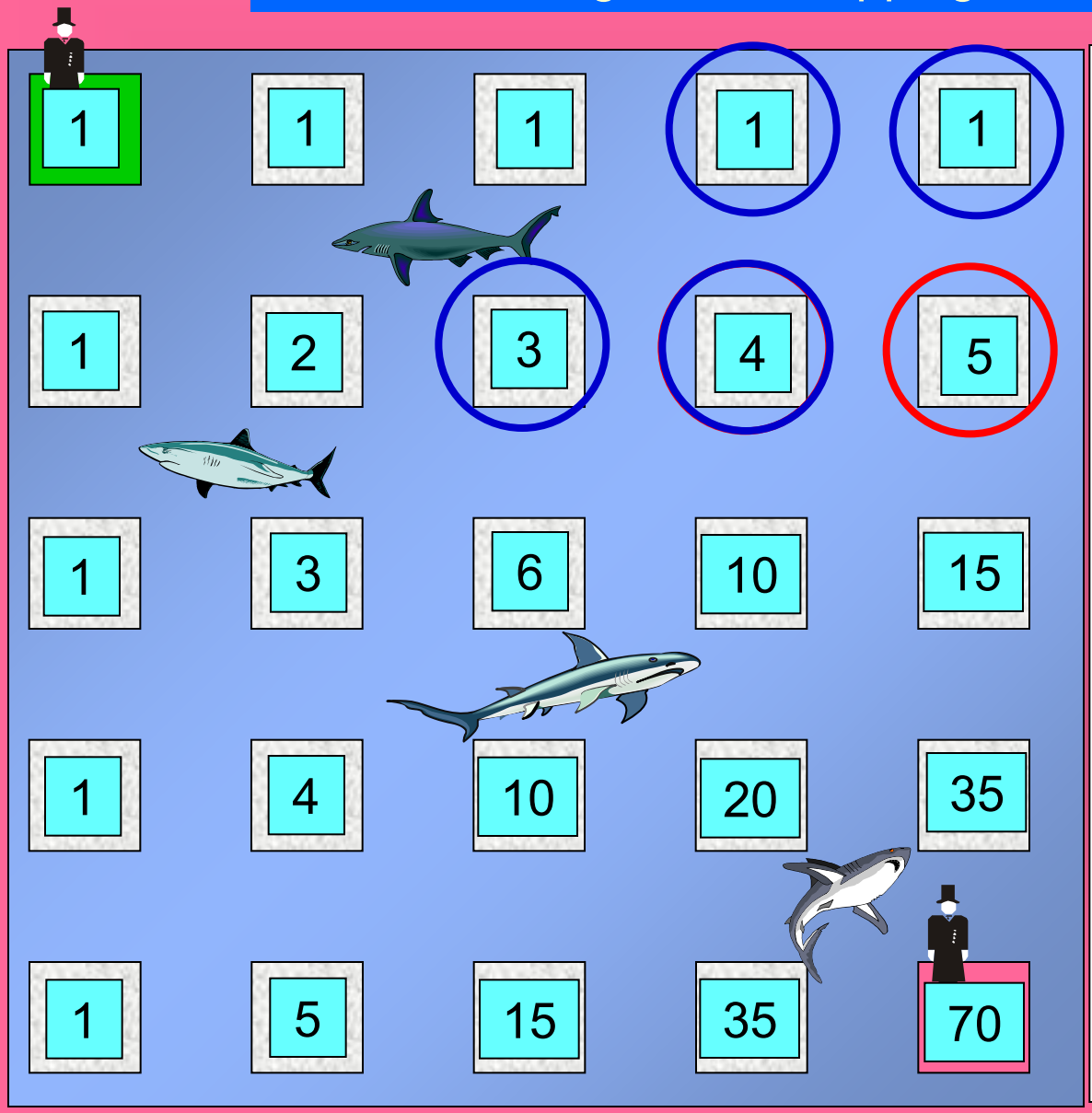


How many routes are there to:

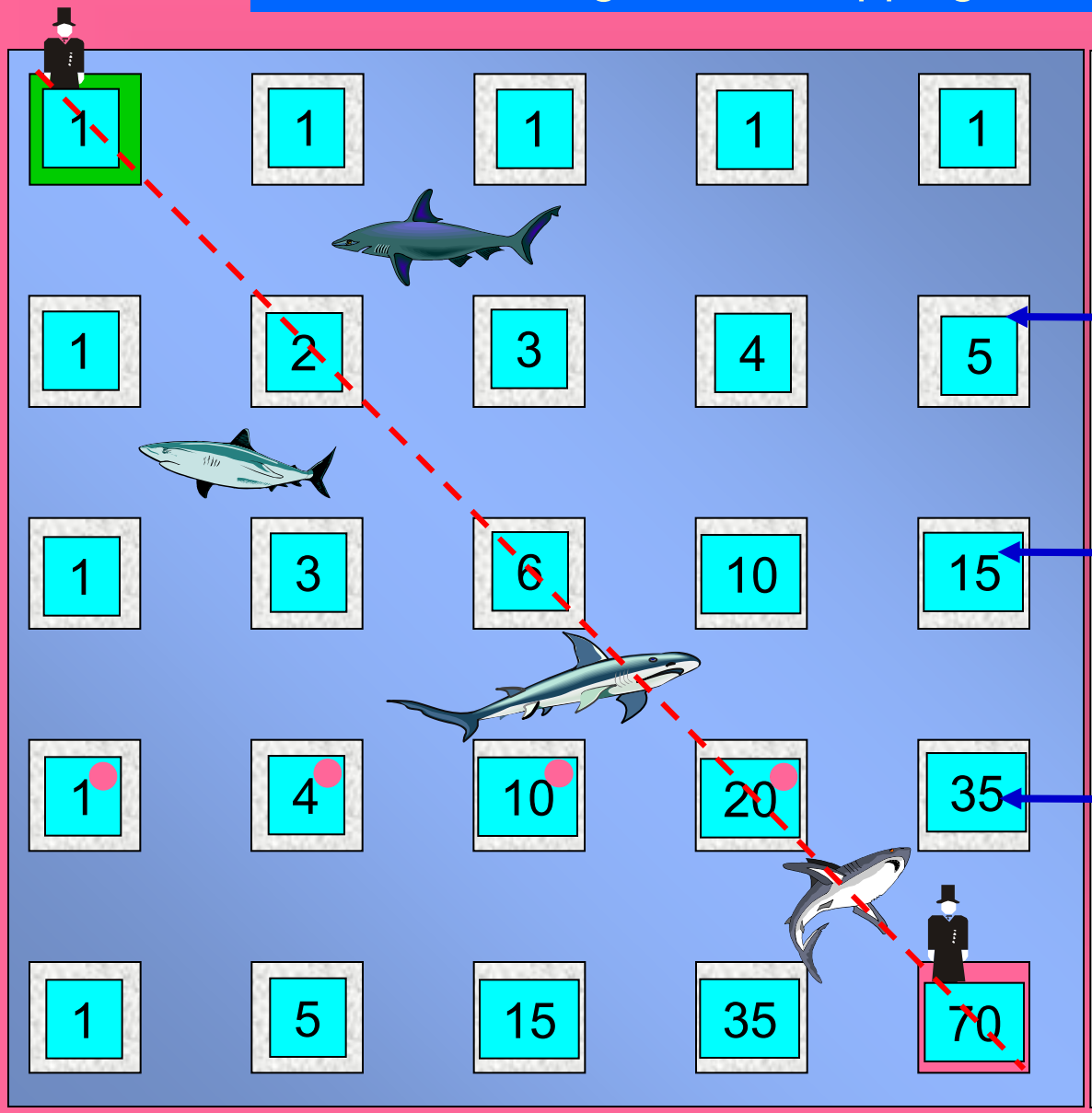
Can you see all 6 of the routes?

How could you have calculated the 6 routes without the need to draw or visualising them?

What do you have to do to get the number of routes to any stone?

Calculate the total number of routes to the finish stone.

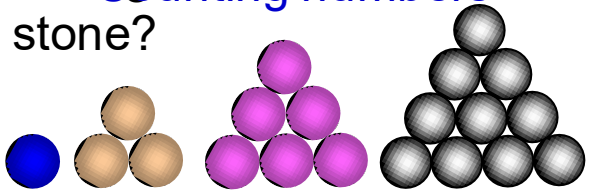
Pascal's Triangle: The Stepping Stone Game



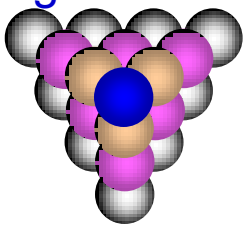
The numbers are symmetrical about the [illegible] produced by the routes through to the finish stone?

Do you notice anything

Counting numbers

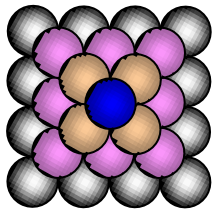


Triangular numbers



Tetrahedral numbers

Square base



Pyramid numbers

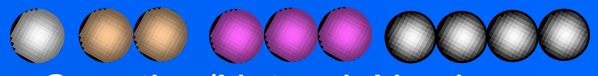
1 5 14 30

Pascal's Triangle

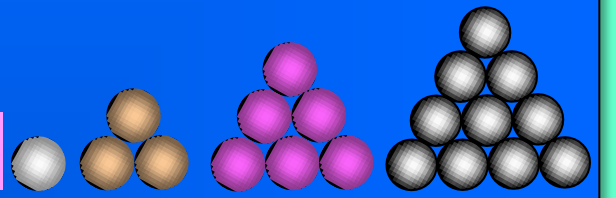
Blaisé Pascal
(1623-1662)

1. Complete the rest of the triangle.
2. Find the sum of each row.
3. Write the sum as a power of 2.

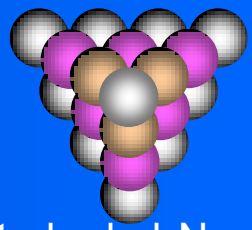
R_0	1											1	$=2^0$
R_1	1	1										2	$=2^1$
R_2	1	2	1									4	$=2^2$
R_3	1	3	3	1								8	$=2^3$
R_4	1	4	6	4	1							16	$=2^4$
R_5	1	5	10	10	5	1						32	$=2^5$
R_6	1	6	15	20	15	6	1					64	$=2^6$
R_7	1	7	21	35	35	21	7	1				128	$=2^7$
R_8	1	8	28	56	70	56	28	8	1			256	$=2^8$
R_9	1	9	36	84	126	126	84	36	9	1		512	$=2^9$
R_{10}	1	10	45	120	210	252	210	120	45	10	1	1024	$=2^{10}$
R_{11}	1	11	55	165	330	462	462	330	165	55	11	1	2048	$=2^{11}$
R_{12}	1	12	66	220	495	792	924	792	495	220	66	12	1	4096	$=2^{12}$
R_{13}	1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1	8192	$=2^{13}$



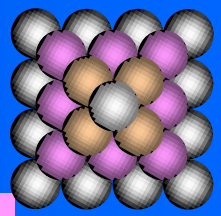
Counting/Natural Numbers



Triangular Numbers



Tetrahedral Numbers

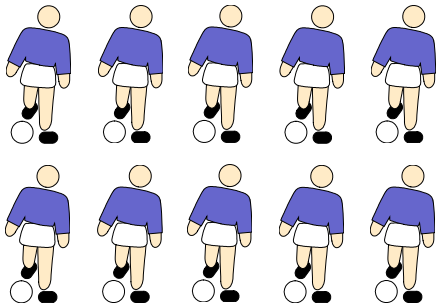


Pyramid Numbers (square base)

^nC_r

R_0	1											
R_1	1	1										
R_2	1	2	1									
R_3	1	3	3	1								
R_4	1	4	6	4	1							
R_5	1	5	10	10	5	1						
R_8	1	8	28	56	70	56	28	8	1			
R_9	1	9	36	84	126	126	84	36	9	1		
R_{10}	1	10	45	120	210	252	210	120	45	10	1	
R_{11}	1	11	55	165	330	462	462	330	165	55	11	1
R_{12}	1	12	66	220	495	792	924	792	495	220	66	12	1
R_{13}	1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1

Choose 0



In how many ways can a 5-a-side team be chosen from a squad of 10 players?

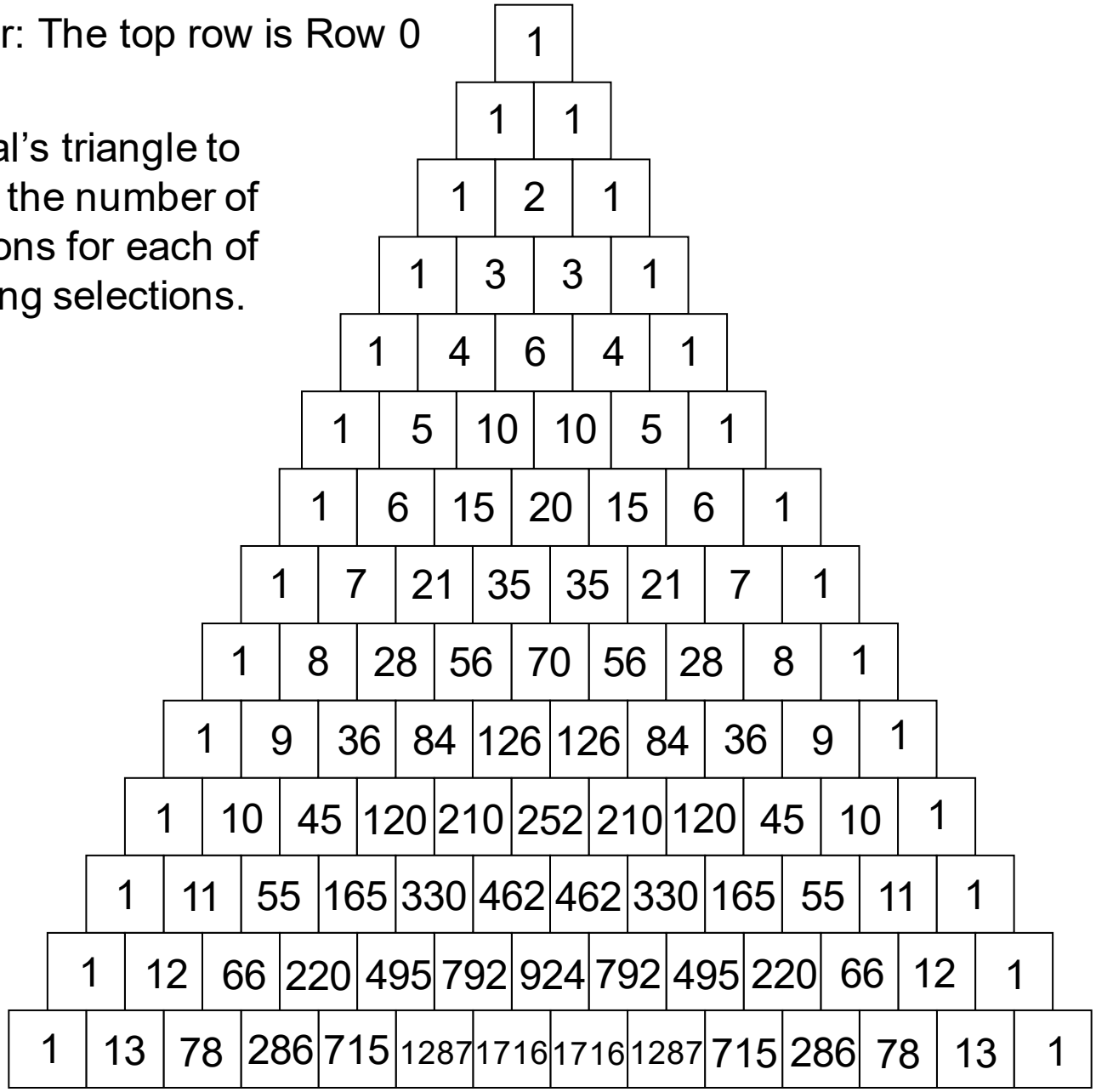
The probability of choosing one particular combination of 5 players is 1/252

252

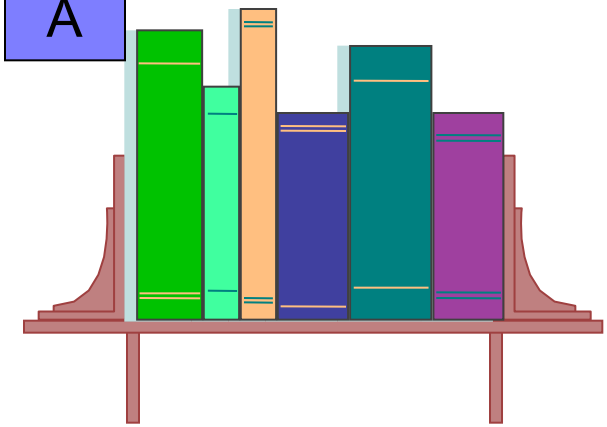
$^{10}C_5$

Remember: The top row is Row 0

Use Pascal's triangle to determine the number of combinations for each of the following selections.

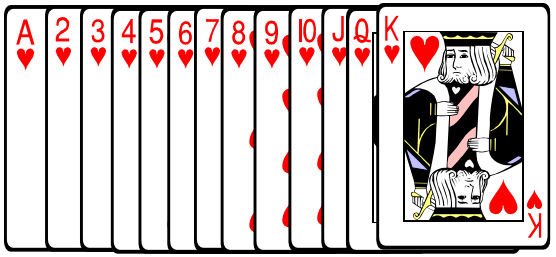


A



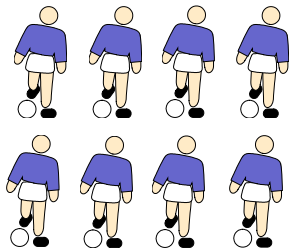
Choose 3 books

C



Choose 7 cards

B



Choose 5 players

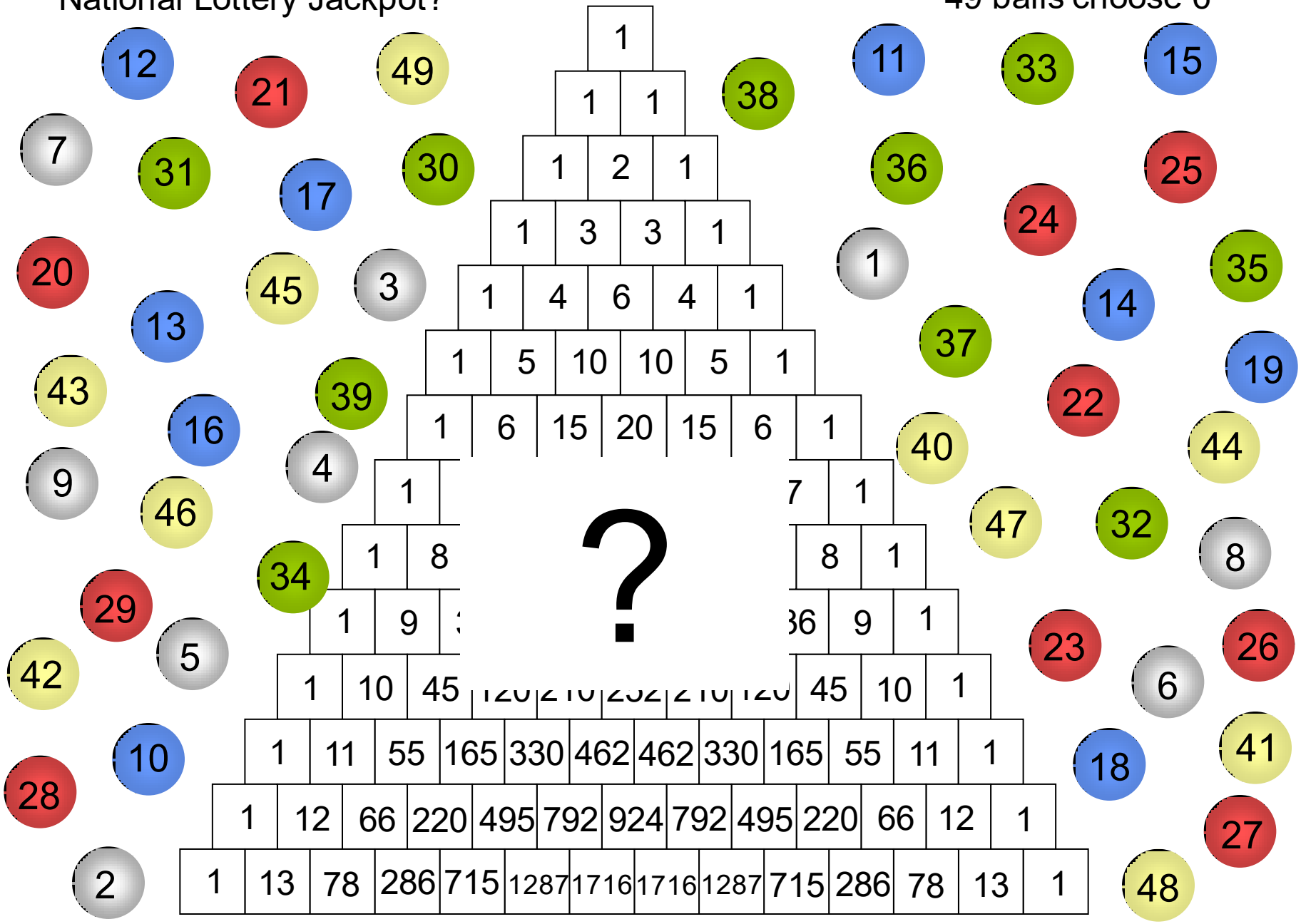
D

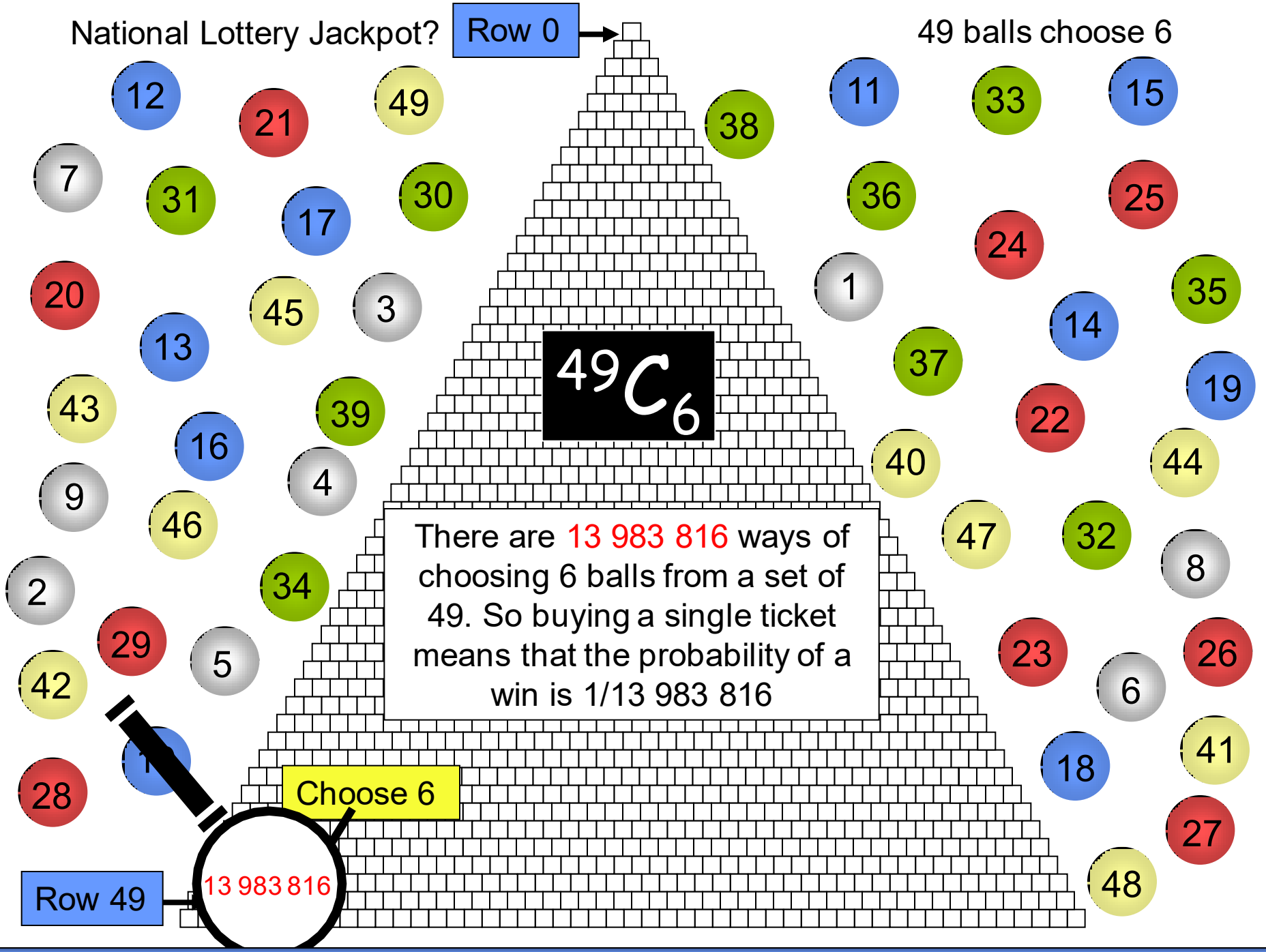
Choose 4 balls

1													
1	1												
1	2	1											
1	3	3	1										
1	4	6	4	1									
1	5	10	10	5	1								
1	6	15	20	15	6	1							
1	7	21	35	35	21	7	1						
1	8	28	56	70	56	28	8	1					
1	9	36	84	126	126	84	36	9	1				
1	10	45	120	210	252	210	120	45	10	1			
1	11	55	165	330	462	462	330	165	55	11	1		
1	12	66	220	495	792	924	792	495	220	66	12	1	
1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1

National Lottery Jackpot?

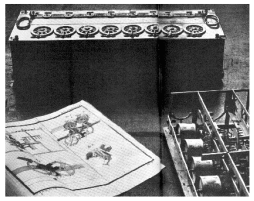
49 balls choose 6



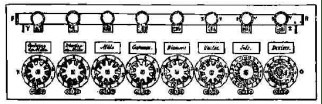

National Lottery Jackpot?
Row 0
49 balls choose 6
12
21
49
38
11
33
15
7
31
30
36
25
17
24
20
45
3
1
35
14
13
37
19
43
39
22
16
40
44
4
9
46
47
32
8
2
34
29
23
26
5
42
6
41
18
28
27
Choose 6
48
Row 49
13 983 816
$^{49}C_6$
There are 13 983 816 ways of choosing 6 balls from a set of 49. So buying a single ticket means that the probability of a win is 1/13 983 816

Historical Note

Blaisé Pascal
(1623-1662)



The Pascaline

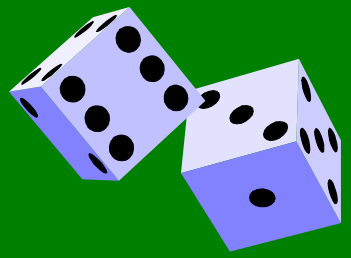


Pierre de Fermat
(1601 – 1675)

Pascal was a French mathematician whose contemporaries and fellow countrymen included Fermat, Descartes and Mersenne. Among his many achievements was the construction of a mechanical calculating machine to help his father with his business. It was able to add and subtract only, but it was a milestone on the road to the age of computers.

He corresponded with Fermat on problems that led to the new branch of mathematics called Probability Theory. The two problems that they examined concerned outcomes when throwing dice and how to divide the stake fairly amongst a group of players if a game was interrupted.

These investigations led Pascal to construct tables of probabilities that eventually led to the triangle of probabilities that bears his name.



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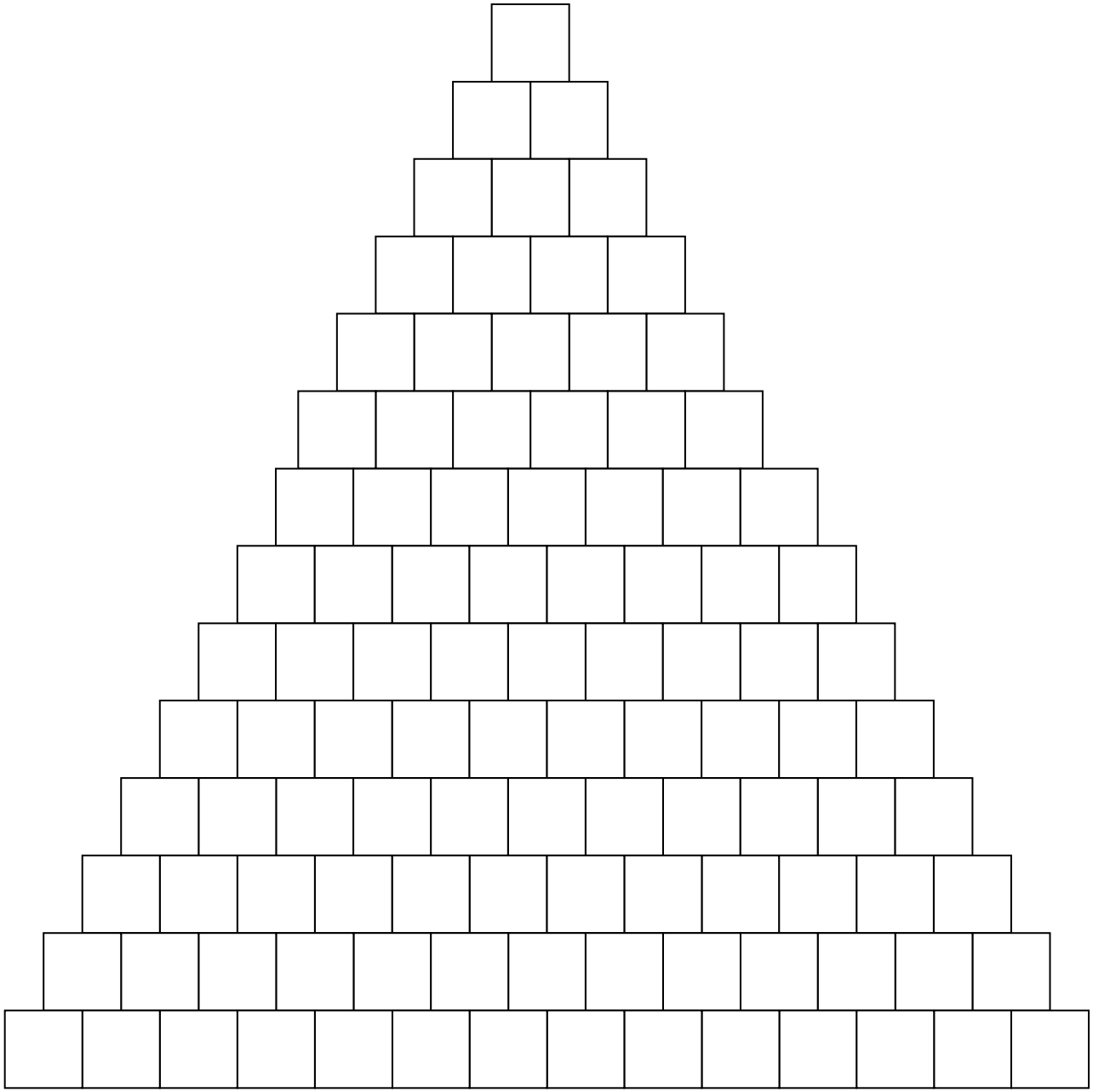

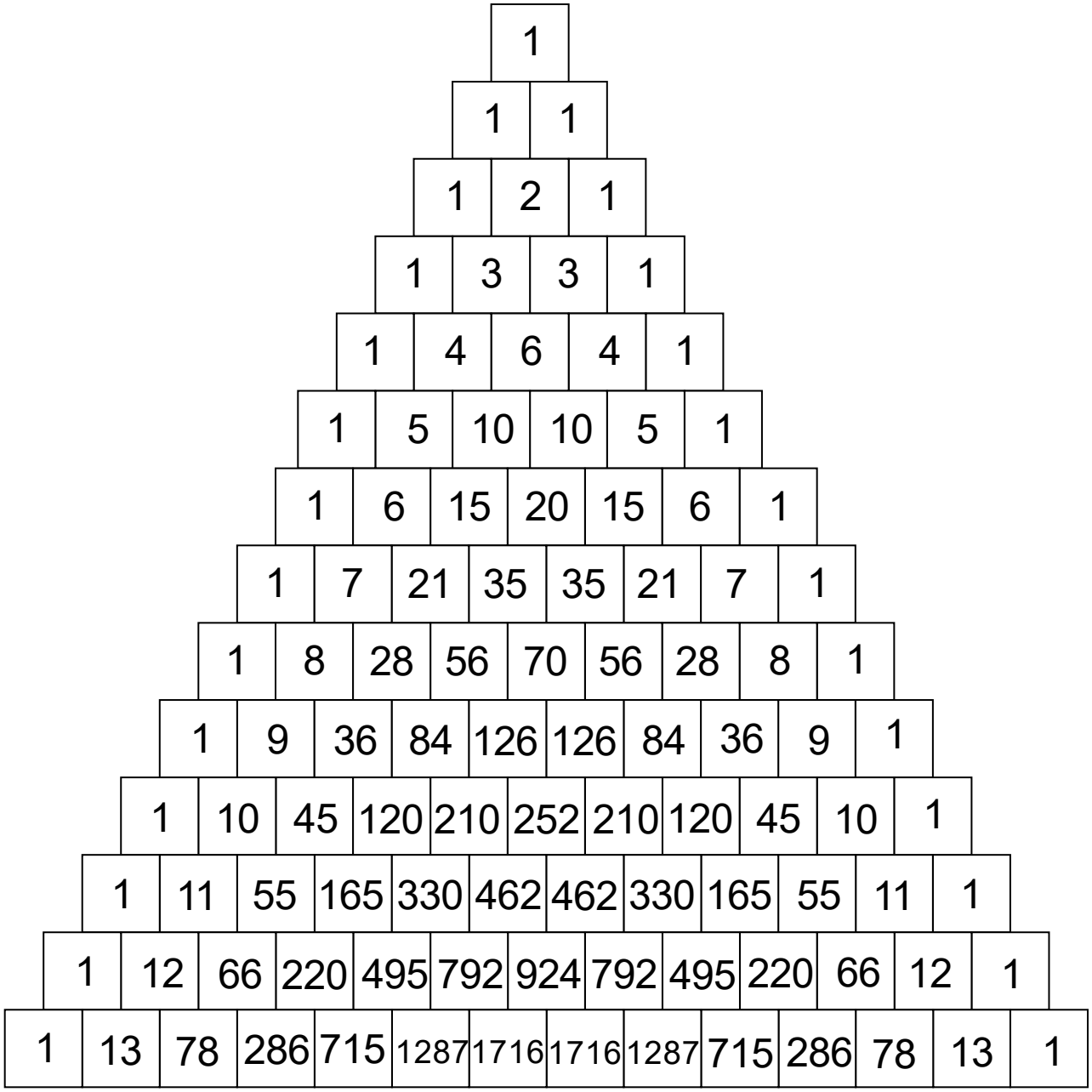
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1