The Fundamentals of Logic

Applications of Propositional Logic

Credit

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Applications of Propositional Logic

- Translating English to Propositional Logic
- System Specifications
- Boolean Searching
- Logic Puzzles
- Logic Circuits

Translating English Sentences

- Steps to convert an English sentence to a statement in propositional logic
  - Identify **atomic propositions** and represent using propositional variables.
  - Determine appropriate **logical connectives**
- "If I go to Riyadh or to Jubail, I will not go shopping."
- "If I go to Riyadh or to Jubail, I will not go shopping."
  - *p:* I go to Riyadh
  - *q:* I go to Jubail
  - *r:* I will go shopping

$s \rightarrow t$ means if s, t

If p or q then not r

$$(p \vee q) \rightarrow \neg r$$

Example

Problem: Translate the following sentence into propositional logic:

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

$s \to t$ means s only if t

A Solution: Let

p represents "You can access the internet from campus",

q represents "You are a computer science major",

r represents "You are a freshman"

$$p \to (q \vee \neg r)$$

System Specifications

- System and Software engineers take requirements in English and express them in a precise specification language based on logic.

Example: Express in propositional logic:

"The automated reply cannot be sent when the file system is full"

"The automated reply cannot be sent when the file system is full"

$u \rightarrow v$ means v when u

One possible solution: Let

p denotes "The automated reply can be sent"

q denotes "The file system is full"

$q \rightarrow \neg p$

Contrapositive of $r \rightarrow s$ is $\neg s \rightarrow \neg r$.

$\neg(\neg p) \rightarrow \neg q$

$p \rightarrow \neg q$

Consistent System Specifications

Definition: A list of propositions is **consistent** if it is possible to assign true/false values to the proposition variables so that each proposition is **TRUE**.

Exercise: Are these specifications consistent?

- "The diagnostic message is stored in the buffer or it is retransmitted." $p \vee q$
- "The diagnostic message is not stored in the buffer." $\neg p$
- "If the diagnostic message is stored in the buffer, then it is retransmitted." $p \to q$
- $p \vee q, \neg p, p \to q$. When p is **FALSE** and q is **TRUE** all three statements are **TRUE**. So the specification is **consistent**.

Boolean Searches

- Logical connectives are used in searches of large collections of information
  - Indexes of Web pages, these searches employ techniques from propositional logic
- AND is used to match records that contain both of the two search items
- OR is used to match one or both of two search items
- NOT is used to exclude a particular search item
  - \- is used in Google

Logic Puzzles

- An island has two kinds of inhabitants, *knights*, **who always tell the truth**, and *knaves*, **who always lie**.
- You go to the island and meet A and B.
  - A says “B is a knight.”
  - B says “The two of us are of opposite types.”

What are the types of A and B?

Logic Puzzles

- A says "B is a knight."
- B says "The two of us are of opposite types."

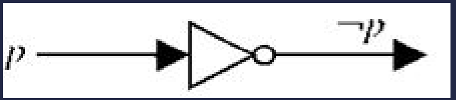
Solution: Let

- p be the statement: A is a knight and
- q be the statement: B is a knight

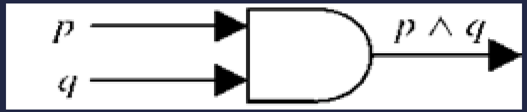
So, then

- $\neg p$ represents the proposition: A is a knave and
- $\neg q$ represents the proposition B is a knave.
  - If A is a knight, then p is true. Since knights tell the truth, q must also be true. This is a contradiction.
  - If A is a knave, then B must not be a knight since knaves always lie. So, then both $\neg p$ and $\neg q$ hold since both are knaves.

Logic Circuits

- Electronic circuits; each input/output signal can be viewed as a 0 or 1.
  - **0** represents **FALSE** and **1** represents **TRUE**
- Complicated circuits are constructed from three basic circuits called gates.
  - The inverter (**NOT gate**) takes an input bit and produces the negation of that bit. 

  - The **OR gate** takes two input bits and produces the value equivalent to the disjunction of the two bits. 

  - The **AND gate** takes two input bits and produces the value equivalent to the conjunction of the two bits. 

Logic Circuits

- More complicated digital circuits can be constructed by combining these basic circuits to produce the desired output.